Journal of King Saud University – Science (2017) **29**, 119–125

King Saud University
Journal of King Saud University – Science
www.ksu.edu.sa
www.sciencedirect.com

ORIGINAL ARTICLE

Effects on magnetic field in squeezing flow of a Casson fluid between parallel plates

Naveed Ahmed [a], Umar Khan [a], Sheikh Irfanullah Khan [a,b], Saima Bano [a], Syed Tauseef Mohyud-Din [a,*]

[a] *Department of Mathematics, Faculty of Sciences, HITEC University, Taxila Cantt, Pakistan*
[b] *COMSATS Institute of Information Technology, University Road, Abbottabad, Pakistan*

Received 5 December 2014; accepted 22 March 2015
Available online 27 March 2015

KEYWORDS

Squeezing flows;
Variation of parameters method (VPM);
Magneto hydrodynamic (MHD);
Parallel plates;
Casson fluid

Abstract Squeezing flow of an electrically conducting Casson fluid has been taken into account. The laws of conservations under the similarity transformation suggested by Wang (1976) have been used to extract a highly nonlinear ordinary differential equation governing the magneto hydrodynamic (MHD) flow. Resulting equation has been solved analytically by using the variation of parameters method (VPM). A RK-4 numerical solution has also been sought to examine the validity of analytical results. Both the solutions are found to be in an excellent agreement. Convergence of the solution is also discussed. Flow behavior under the modifying involved physical parameters is also discussed and explained in detail with the graphical aid. It is observed that magnetic field can be used as a control phenomenon in many flows as it normalizes the flow behavior. Also, squeeze number plays an important role in these types of problems and an increase in squeeze number increases the velocity profile.

1. Introduction

Many mechanical equipment work under the principle of moving pistons where two plates exhibit the squeezing movement normal to their own surfaces. Electric motors, engines and hydraulic lifters also have this clutching flow in some of their parts. Due to this practical significance squeezing flow between parallel plates has become one of the most active research fields in fluid mechanics. Its biological applications are also of equal importance. Flow inside syringes and nasogastric tubes is also a kind of squeezing flows.

Foundational work regarding squeezing flows can be named to Stefan (1874) who presented basic formulation of these types of flows under lubrication assumption. After him numbers of scientist have shown their interests toward squeezing flows and have carried out many scientific studies to understand these flows. Some of selected contributions are mentioned in forthcoming lines.

1986 Reynolds (1886) investigated the squeezing flow between elliptic plates. Archibald (1956) considered the same

* Corresponding author.
E-mail address: syedtauseefs@hotmail.com (S.T. Mohyud-Din).
Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jksus.2015.03.006
1018-3647

problem for rectangular plates. After that several attempts have been made by different researchers to understand squeezing flows in a better way (Grimm, 1976; Wolfe, 1965; Kuzma, 1968; Tichy and Winer, 1970; Jackson, 1962).

Earlier studies on squeezing flows were based on Reynolds equation whose insufficiency for some cases has been shown by Jackson (1962) and Usha and Sridharan (1996). Due to efforts of Birkhoff (1960), Yang (1958) and Wang and Watson (1979) more flexible and useful similarity transforms are now available. These similarity transforms reduce the Navier–Stokes equation into a fourth order nonlinear ordinary differential equation and have further been used in some other investigations as well (Wang, 1976; Laun et al., 1999; Hamdan and Baron, 1992; Nhan, 2000; Rashidi et al., 2008).

Flow of electrically conducting non-Newtonian fluid is a very important phenomenon as in most of the practical situations we have to deal with the flow of conducting fluid which exhibits different behaviors under the influence of magnetic forces. In these cases magneto hydro dynamic (MHD) aspect of the flow is also needed to be considered. Homotopy perturbation solution for Two-dimensional MHD squeezing flow between parallel plates has been determined by Siddiqui et al. (2008). Domairry and Aziz (2009) investigated the same problem for the flow between parallel disks. Recently, Mustafa et al. (2012) examined heat and mass transfer for squeezing flow between parallel plates using the homotopy analysis method (HAM).

In most of realistic models the fluids involved are not simple Newtonian. Complex rheological properties of non-Newtonian fluids cannot be captured by a single model. Different mathematical models have been used to study different types of non-Newtonian fluids. One of such models is known as Casson fluid model. (Mrill et al., 1965; McDonald, 1974) showed that it is the most compatible formulation to simulate blood type fluid flows. It is clear from the literature survey that the squeezing flow of a Casson fluid between the plates moving normal to their own surface is yet to be inspected.

Due to the inherent nonlinearity of the equations governing the fluid flow exact solutions are very rare. Even where they are available immense simplification assumptions have been imposed. Those overly imposed suppositions may not be used for more realistic flows. However to deal with this hurdle many analytical approximation techniques have been developed which are commonly used nowadays (Abbasbandy, 2007a; Abbasbandy, 2007b; Abdou and Soliman, 2005; Noor and Mohyud-Din, 2007; Asadullah et al., 2013; Khan et al., 2012; Ahmed et al., 2014). Variation of parameters method (VPM) is one of these recently developed analytical techniques that have been used to solve different problems (Khan et al., 2014; Noor et al., 2008; Mohyud-Din et al., 2009; Khan et al., 2014a; Khan et al., 2014b; Khan et al., 2014c; Zaidi et al., 2013).

A literature survey reveals that no attempt has ever been made to study the MHD squeezing flow of a Casson fluid. So, in this paper we have presented a comprehensive study for this problem. VPM has been applied to study the nonlinear ordinary differential equation. A numerical solution to the problem has also been sought by using the Runge Kutta order 4 method. Comparison between both the solutions shows that the results obtained by VPM are in excellent agreement with the numerical results.

2. Governing equations

We consider an incompressible flow of a Casson fluid between two parallel plates separated by a distance $z = \pm l(1-\alpha t)^{1/2} = \pm h(t)$, where l is the initial gap between the plates (at a time $t = 0$). Additionally $\alpha > 0$ corresponds to a squeezing motion of both the plates until they touch each other at $t = 1/\alpha$, for $\alpha < 0$ the plates bear a receding motion and dilate as described in Fig. a. Rheological equation for Casson fluid is defined as under (Nadeem et al., 2012; Nadeem et al., 2013; Nadeem et al., 2014a; Nadeem et al., 2014b; Nadeem et al., 2014c; Ahmed et al., 2013; Casson, 1959; Akber and Khan, 2015; Akbar et al., 2014; Nakamura and Sawada, 1987; Nakamura and Sawada, 1988)

$$\tau_{ij} = \begin{Bmatrix} 2\left[\mu_B + \left(\frac{p_y}{2\pi}\right)\right]e_{ij}, & \pi > \pi_c \\ 2\left[\mu_B + \left(\frac{p_y}{2\pi_c}\right)\right]e_{ij}, & \pi_c > \pi \end{Bmatrix} \tag{1}$$

where $\pi = e_{ij}e_{ij}$, and e_{ij} is the (i, j)th component of the deformation rate, i.e. π is the product of the component of deformation rate with itself, π_c is the critical value of the said product, μ_B is plastic dynamic viscosity of the non-Newtonian fluid and p_y is yield stress of slurry fluid.

A constant magnetic field of strength M_0 is imposed perpendicular and relatively fixed to the walls. We are also applying the following assumptions on the flow model:

(a) The effects of induced magnetic and electric fields produced due to the flow of electrically conducting fluid are negligible.
(b) No external electric field is present.

Under aforementioned constraints the conservation equations for the flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(1+\frac{1}{\gamma}\right)\left(2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right) - \frac{\sigma\beta^2}{\rho}u, \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(1+\frac{1}{\gamma}\right)\left(2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y \partial x}\right), \end{aligned} \tag{3}$$

where u and v are the velocity components in x and y-directions respectively, p is the pressure, $\upsilon = \frac{\mu}{\rho}$ is the dynamic viscosity of the fluid (ratio of kinematic viscosity and density),

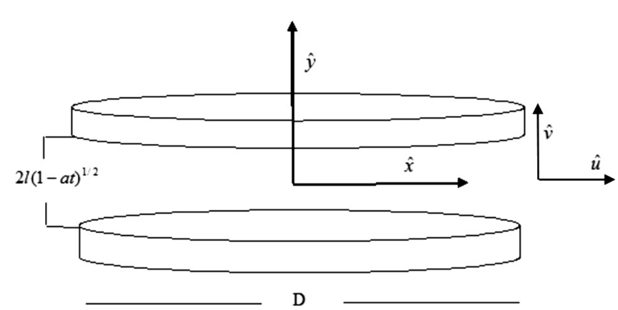

Figure a Schematic diagram for the flow problem.

$\gamma = \mu_{\mathbf{B}}\sqrt{2\pi_c}/p_y$ is the Casson fluid parameter and β is the magnitude of imposed magnetic field.

Supporting conditions for the problem are as follows

$$\begin{aligned} &u = 0, \quad v = v_w = \tfrac{dh}{dt} \quad at\ y = h(t), \\ &\tfrac{\partial u}{\partial y} = 0, \quad v = 0 \qquad at\ y = 0. \end{aligned} \tag{4}$$

We can simplify the above system of equations by eliminating the pressure terms from Eqs. (2) and (3) and using Eq. (1). After cross differentiation and introducing vorticity ω we get

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial \hat{x}} + v\frac{\partial \omega}{\partial \hat{y}} = \upsilon\left(1+\frac{1}{\gamma}\right)\left(\frac{\partial^2\omega}{\partial \hat{x}^2} + \frac{\partial^2\omega}{\partial \hat{y}^2}\right) - \frac{\sigma\beta^2}{\rho}\frac{\partial u}{\partial y}, \tag{5}$$

where,

$$\omega = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{6}$$

Transform introduced by Wang (1976) for a two-dimensional flow is sated as

$$u = \frac{\alpha x}{[2(1-\alpha t)]}F'(\eta), \tag{7}$$

$$v = \frac{-\alpha l}{[2(1-\alpha t)^{1/2}]}F(\eta), \tag{8}$$

where,

$$\eta = \frac{y}{[l(1-\alpha t)^{1/2}]}. \tag{9}$$

Substituting Eqs. (7)–(9) in Eq. (5) using (6), we obtain a nonlinear ordinary differential equation describing the Casson fluid flow as

$$\begin{aligned} &(1+\frac{1}{\gamma})F^{iv}(\eta) - S(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) \\ &\quad - F(\eta)F'''(\eta)) - M^2F''(\eta) = 0, \end{aligned} \tag{10}$$

where $S = \alpha l^2/2\nu$ denotes the non-dimensional Squeeze number. Using Eqs. (7)–(9) boundary conditions for the problem reduce to

$$F(0) = 0, \quad F''(0) = 0, F(1) = 1, \quad F''(1) = 0. \tag{11}$$

Here, squeeze number S describes the movement of the plates ($S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to collapsing movement of the plates). It is pertinent to mention here that for $M = 0$ and $\gamma \to \infty$, our study reduces to the one obtained by Wang (1976).

Skin friction coefficient is defined as

$$C_f = \nu\left(1+\frac{1}{\gamma}\right)\frac{\left(\frac{\partial u}{\partial y}\right)_{y=h(t)}}{v_w^2}.$$

In terms of Eqs. (7)–(9), we have

$$l^2/x^2(1-at)\mathrm{Re}_x C_f = \left(1+\frac{1}{\gamma}\right)F'(1). \tag{12}$$

Here $\mathrm{Re}_x = \frac{2lv_w^2}{\nu x(1-at)^{1/2}}$

3. Solution procedure

Using the standard procedure for VPM (Noor et al., 2008; Mohyud-Din et al., 2009; Khan et al., 2014a; Khan et al., 2014b; Khan et al., 2014c; Zaidi et al., 2013), we can write Eq. (10) as

$$\begin{aligned} F_{n+1}(\eta) = A_1 + A_2\eta + A_3\frac{\eta^2}{2} + A_4\frac{\eta^3}{6} - \int_0^\eta \left(\frac{\eta^3}{3!} - \frac{\eta^2 s}{2!} + \frac{\eta s^2}{2!} + \frac{s^3}{3!}\right) \\ \times \left(-\left(\frac{\gamma}{1+\gamma}\right)\begin{pmatrix} (S)\begin{pmatrix} sF(s) + 3F''(s) + F'(s)F''(s) \\ -F(s)F'''(s)\end{pmatrix} \\ -M^2F''(s)\end{pmatrix}\right)ds, \end{aligned} \tag{13}$$

Here, A_1, A_2, A_3 and A_4 are the constants obtained by taking the highest order linear term of the Eq. (10) and integrating it 4 times to get the final form of the scheme. Using the boundary conditions given in Eq. (11), the above equation can be written as

$$\begin{aligned} F_{n+1}(\eta) = A_2\eta + A_4\frac{\eta^3}{6} - \int_0^\eta \left(\frac{\eta^3}{3!} - \frac{\eta^2 s}{2!} + \frac{\eta s^2}{2!} + \frac{s^3}{3!}\right) \\ \times \left(-\left(\frac{\gamma}{1+\gamma}\right)\begin{pmatrix} S\begin{pmatrix} sF(s) + 3F''(s) + F'(s)F''(s) \\ -F(s)F'''(s)\end{pmatrix} \\ -M^2F''(s)\end{pmatrix}\right)ds, \end{aligned} \tag{14}$$

with $n = 0, 1, 2, \ldots$, where A_2 and A_4 are the constants which can be computed by using the boundary conditions $F(1) = 1$ and $F'(1) = 0$, respectively.

First few terms of the solution are given as

$$\begin{aligned} &F_0(\eta) = \left(\tfrac{\gamma}{1+\gamma}\right)\left(A_2\eta + A_4\tfrac{\eta^3}{6}\right), \\ &F_1(\eta) = \left(\tfrac{\gamma}{1+\gamma}\right)\left(A_2\eta + A_4\tfrac{\eta^3}{6}\right) - \left(\tfrac{\gamma}{1+\gamma}\right)\left(\tfrac{1}{30}SA_4 + \tfrac{1}{120}M^2A_4\right)\eta^5 \\ &-\left(\tfrac{\gamma}{1+\gamma}\right)\left(\tfrac{1}{2520}SA_4^2\right)\eta^7, \\ &F_2(\eta) = \left(\tfrac{\gamma}{1+\gamma}\right)\left(A_2\eta + A_4\tfrac{\eta^3}{6}\right) - \left(\tfrac{\gamma}{1+\gamma}\right)\left(\tfrac{1}{30}SA_4 + \tfrac{1}{120}M^2A_4\right)\eta^5 \\ &-\left(\tfrac{\gamma}{1+\gamma}\right)\begin{pmatrix} \tfrac{1}{2520}SA_4^2 + \tfrac{1}{210}S^2A_4 - \tfrac{1}{2520}SM^2A_4A_2 \\ -\tfrac{1}{630}S^2A_2A_4 + \tfrac{1}{504}SM^2A_4 + \tfrac{1}{5040}M^4A_4 \end{pmatrix}\eta^7 \\ &-\left(\tfrac{\gamma}{1+\gamma}\right)\left(\tfrac{1}{11340}S^2A_4^2 - \tfrac{1}{45360}S^2A_2A_4^2 + \tfrac{1}{60480}M^2SA_4^2\right)\eta^9 \\ &-\left(\tfrac{\gamma}{1+\gamma}\right)\left(\tfrac{1}{178200}S^3A_4^2 - \tfrac{1}{2494800}S^2A_4^3 + \tfrac{1}{356400}S^2M^2A_4^2 + \tfrac{1}{2851200}SM^4A_4^2\right)\eta^{11} + \ldots . \end{aligned} \tag{15}$$

Similarly, the other iterations for the solution can also be computed.

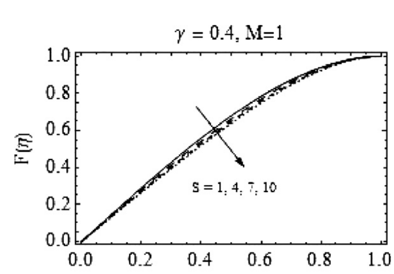

Figure 1 Effects of positive values of S on $F(\eta)$.

4. Results and discussions

In this section the influences of squeeze number S, Casson fluid parameter γ and the magnetic number M on the axial $(F(\eta))$and radial $(F^{'}|(\eta))$ velocities are characterized. Figs. 1–16 are displayed for the said purpose. First eight figures are for the case when the plates are moving apart ($S > 0$). Fig. 1 depicts the effects of increasing values of squeeze number Son the axial velocity $F(\eta)$. It is clear, increasing S results in a decreased axial velocity. Effects of rising S on radial velocity are shown in Fig. 2. For increasing S, a decrease in $F^{'}(\eta)$ is observed when $\eta \leqslant 0.5$. However, there is an increase in $F^{'}(\eta)$ for $0.5 < \eta \leqslant 1$. Fig. 3 illustrates the behavior of Casson fluid parameter γ on $F(\eta)$. Increase in γ decelerates the axial flow. Effects of growing γ on radial velocity are shown in Fig. 4. Increasing γ decreases $F^{'}(\eta)$ for $\eta \leqslant 0.5$ and a rise in $F^{'}(\eta)$ is observed for $0.5 < \eta \leqslant 1$.

In Figs. 5–8, the consequences of varying magnetic number M on $F(\eta)$ and $F^{'}(\eta)$ are portrayed. It can be observed from Fig. 5 that for increasing M, there is a decrease in $F(\eta)$ for slightly lower values of squeeze number S; while for $F'(\eta)$, the increase in M provides a velocity profile similar to the case of increasing S. That is, increasing the magnitude of magnetic field fallouts in a uniform decrease in the velocity. Figs. 7 and 8 are drawn to analyze the effects of magnetic field for slightly higher values of squeeze number S. The behavior of axial and radial velocities remains almost similar to lower S.

Figs. 9–16 are presented to study the effects of physical parameters for dilating plates ($S < 0$). In Fig. 9, considerable axial acceleration is observed for falling S. Fig. 10 demonstrates the effects of decreasing squeeze number on radial velocity. It is clear that $F^{'}(\eta)$ increases with squeeze rate for $\eta \leqslant 0.4$. A sudden change in $F^{'}(\eta)$ is observed when $0.4 < \eta \leqslant 1$, i.e. for decreasing values of squeeze number there is a rapid decrease in radial velocity of the fluid. Figs. 11 and 12 show the effects of Casson fluid parameter on axial and

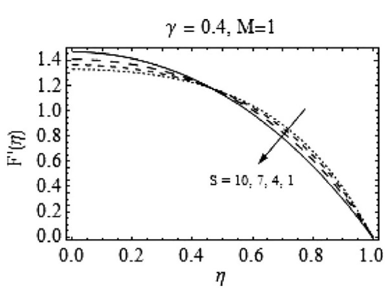

Figure 2 Effects of positive values of S on $F'(\eta)$.

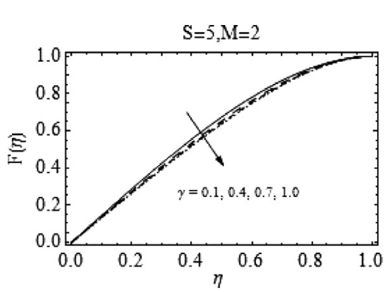

Figure 3 Effects of γ on $F(\eta)$.

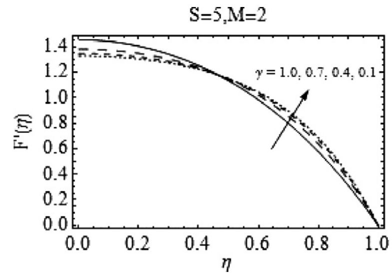

Figure 4 Effects of γ on $F'(\eta)$.

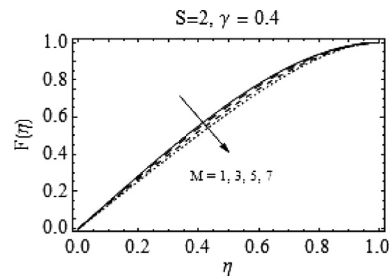

Figure 5 Effects of Mon $F(\eta)$.

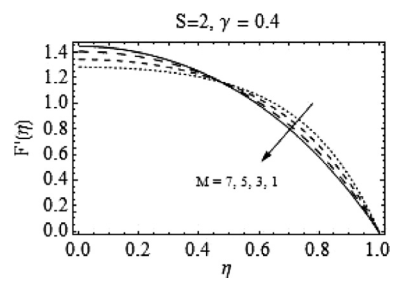

Figure 6 Effects of Mon $F'(\eta)$.

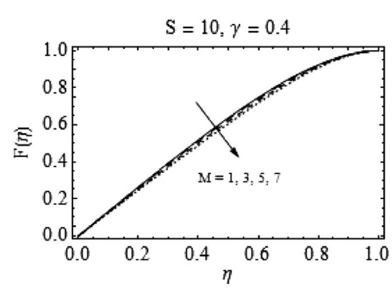

Figure 7 Effects of Mon $F(\eta)$.

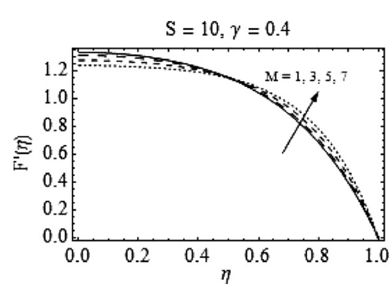

Figure 8 Effects of Mon $F'(\eta)$.

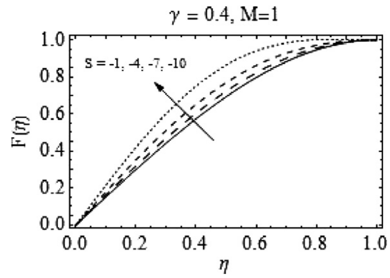

Figure 9 Effects of negative values of S on $F(\eta)$.

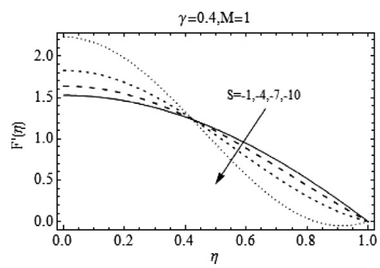

Figure 10 Effects of negative values of S on $F'(\eta)$.

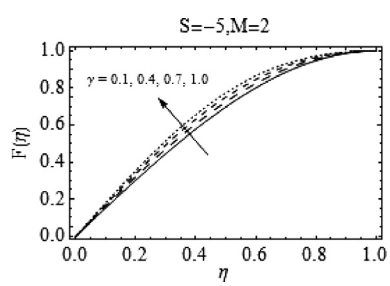

Figure 11 Effects of γ on $F(\eta)$.

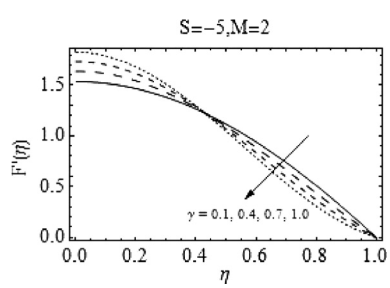

Figure 12 Effects of γ on $F'(\eta)$.

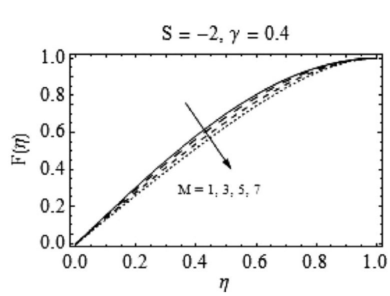

Figure 13 Effects of M on $F(\eta)$.

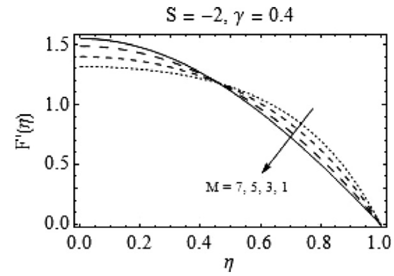

Figure 14 Effects of M on $F'(\eta)$.

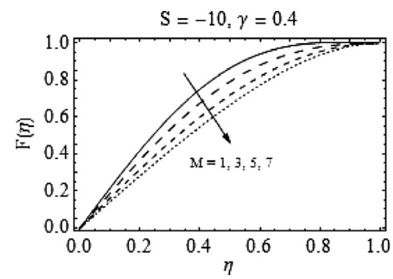

Figure 15 Effects of M on $F(\eta)$.

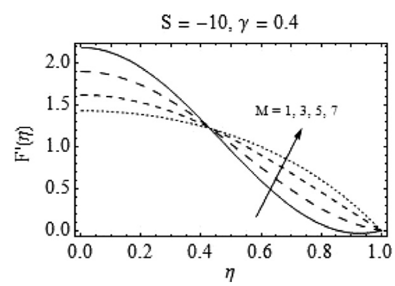

Figure 16 Effects of M on $F'(\eta)$.

Table 1 Convergence of VPM solution for $\gamma = 0.4$ and M = 1.

Order of Approximations	S = 5		S = −5	
	$A_2 = f'$	$A_4 = f'''$	$A_2 = f'$	$A_4 = f'''$
1	1.378126	−1.526101	1.797777	−6.800000
2	1.351366	−1.331071	1.730490	−6.059060
3	1.347469	−1.308010	1.733885	−6.092031
4	1.347157	−1.306332	1.733850	−6.091734
5	1.347136	−1.306259	1.733850	−6.091732
6	1.347136	−1.306257	1.733850	−6.091732
8	1.347136	−1.306257	1.733850	−6.091732
10	1.347136	−1.306257	1.733850	−6.091732
Numerical (RK-4)	1.347136	−1.306257	1.733850	−6.091732

radial velocities respectively. Almost an identical behavior is observed for Casson fluid parameter $\gamma$and the squeeze number S when plates are coming together.

Figs. 13–16 present behavior of the flow when plates are coming together ($S < 0$) and the magnetic number is varying. In Fig. 13, the effects of increasing magnetic number on $F(\eta)$ are displayed and a decrease in $F(\eta)$ is observed for slightly

Table 2 Comparison between the numerical and VPM solutions for $\gamma = 0.4$ and M = 1.

η	$S = 5$				$S = -5$			
	$F(\eta)$ VPM	Numerical	$F'(\eta)$ VPM	Numerical	$F(\eta)$ VPM	Numerical	$F'(\eta)$ VPM	Numerical
0	0	0	1.359393	1.359393	0	0	1.677216	1.677216
0.1	0.139081	0.139081	1.348452	1.348452	0.166839	0.166839	1.650804	1.650804
0.2	0.276358	0.276358	1.357517	1.357517	0.328444	0.328444	1.572994	1.572994
0.3	0.409918	0.409918	1.310148	1.310148	0.479861	0.479861	1.447971	1.447971
0.4	0.537628	0.537628	1.239953	1.239953	0.616685	0.616685	1.282424	1.282424
0.5	0.657014	0.657014	1.142869	1.142869	0.735286	0.735286	1.085120	1.085120
0.6	0.765125	0.765125	1.013414	1.013414	0.832992	0.832992	0.866366	0.866366
0.7	0.858383	0.858383	0.844480	0.844480	0.908218	0.908218	0.637365	0.637365
0.8	0.932408	0.932408	0.627096	0.627096	0.960506	0.960506	0.409532	0.409532
0.9	0.981819	0.981819	0.350136	0.350136	0.990529	0.990529	0.193804	0.193804
1.0	1	0	0	0	1	1	0	0

Table 3 Comparison of VPM and numerical solutions with existing results for $M = 0$ and $\gamma \to \infty$.

$S\downarrow$	Present results (HPM)	Present results (RK-4)	Wang (1976)
−0.9780	−2.1915	−2.1915	−2.235
−0.4977	−2.6193	−2.6193	−2.6272
−0.09998	−2.9277	−2.9277	−2.9279
0	−3.000	−3.000	−3.000
0.09403	−3.0663	−3.0663	−3.0665
0.4341	−3.2943	−3.2943	−3.2969
1.1224	−3.708	−3.708	−3.714

Table 4 Numerical values for skin friction coefficient.

S	γ	M	$\left(1+\frac{1}{\gamma}\right)F''(1)$
−5.0	0.4	1.0	−6.298708
−3.0			−8.320727
−1.0			−9.970376
1.0			−11.376240
3.0			−12.610669
5.0			−13.718095
−3.0	0.1		−30.991005
	0.3		−10.873387
	0.5		−6.771549
3.0	0.1		−35.260196
	0.3		−15.149577
	0.5		−11.078736
−3.0	0.4	2	−13.101572
		4	−14.908219
		6	−17.501183
3.0	0.4	2	−9.038196
		4	−11.531983
		6	−14.819321

higher values of S. Fig. 14 gives us a graphical demonstration of $F'(\eta)$ for growing magnetic number. It shows $F'(\eta)$ decreases for $\eta \leqslant 0.4$ however for $0.4 < \eta \leqslant 1$ it behaves otherwise, i.e. for increasing values of magnetic number, there is a rapid increase in radial velocity of the fluid. A similar type of behavior is observed for increasing magnetic number when S = -10 with more prominent effects. Similarly, in Fig. 16, a quite rapid change can be observed for increasing values of magnetic number. Also, with a decrease in squeeze number the backflow may emerge and a strong magnetic field is required to enhance the flow as shown in Fig. 16.

It is important to check the convergence of the series solution obtained in Eq. (15). For this purpose, the numerical values of unknown constants A_2 and A_4 are computed in Tables 1. It is pertinent to mention that VPM converges only after6 approximations. Obtained analytical results are also compared with the ones acquired by using the RK-4 method and an excellent agreement is found.

A comparison between the analytical and numerical solutions is shown in Table 2 for the axial and radial velocities. It can clearly be seen that the solutions agree very well. Table 3 compares the present results with some already existing solutions in the literature and again an excellent agreement is observed.

Table 4 presents the numerical values of skin friction coefficient. It can be seen that for all increasing parameters an increase in the magnitude of skin friction coefficient is obtained.

5. Conclusions

MHD squeezing flow of a non-Newtonian namely Casson fluid is presented between parallel plates. Governing equations are reduced to a single ordinary differential equation using a similarity transform. Two cases are considered, i.e. one when plates are moving apart and other when plates are coming closer. VPM is used to solve the equations governing the flow. Effects of emerging parameters on flow are demonstrated graphically and a comprehensive discussion is provided. A numerical solution is also obtained using RK-4 method to compare the results obtained by VPM and an excellent agreement is found among the solutions. It can be concluded from the graphs that a strong magnetic field can be used to enhance the flow when plates are coming together and squeeze number increases the velocity profile for both the cases, i.e., when plates are coming closer and when plates are going apart.

References

Abbasbandy, S., 2007a. A new application of He's variational iteration method for quadratic Riccati differential equation by using A domian's polynomials. J. Comput. Appl. Math. 207, 59–63.

Abbasbandy, S., 2007b. Numerical solutions of nonlinear Klein-Gordon equation by variational iteration method. Int. J. Numer. Meth. Energy 70, 876–881.

Abdou, M.A., Soliman, A.A., 2005. Variational iteration method for solving Burger's and coupled Burger's equations. J. Comput. Appl. Math. 181, 245–251.

Ahmed, N., Khan, U., Yang, X.J., Khan, S.I.U., Zaidi, Z.A., Mohyud-Din, S.T., 2013. Magneto hydrodynamic (MHD) squeezing flow of a Casson fluid between parallel disks. Int. J. Phys. Sci. 8 (36), 1788–1799.

Ahmed, N., Khan, U., Zaidi, Z.A., Jan, S.U., Waheed, A., Mohyud-din, S.T., 2014. MHD flow of an incompressible fluid through porous medium between dilating and squeezing permeable walls. J. Porous Media 17 (10), 861–867.

Akbar, N.S., Khan, Z.H., Nadeem, S., 2014. Peristaltic impulsion of MHD biviscosity fluid in a lopsided channel: closed form solution. Eur. Phys. J. Plus 129, 123.

Archibald, F.R., 1956. Load capacity and time relations for squeeze films. J. Lubr. Technol. 78, A231–A245.

Asadullah, M., Khan, U., Ahmed, N., Manzoor, R., Mohyud-Din, S.T., 2013. MHD flow of a Jeffery fluid in converging and diverging channels. Int. J. Modern Math. Sci. 6, 92–106.

Birkhoff, G., 1960. Hydrodynamics, a Study in Logic, Fact and Similitude, Revised ed. Princeton University Press, 137.

Casson, N., 1959. A flow equation for pigment-oil suspension of the printing ink-type. In: Rheology of Disperse Systems. Pergamon, London, 84–104.

Domairry, G., Aziz, A., 2009. Approximate analysis of MHD squeeze flow between two parallel disks with suction or injection by homotopy perturbation method. Math. Prob. Eng., 603916.

Grimm, R.J., 1976. Squeezing flows of Newtonian liquid films: an analysis include the fluid inertia. Appl. Sci. Res. 32 (2), 149–166.

Hamdan, M.H., Baron, R.M., 1992. Analysis of the squeezing flow of dusty fluids. Appl. Sci. Res. 49, 345–354.

Jackson, J.D., 1962. A study of squeezing flow. Appl. Sci. Res. A 11, 148–152.

Khan, Y., Wu, Q., Faraz, N., Yildirim, A., Mohyud-Din, S.T., 2012. Heat transfer analysis on the magnetohydrodynamic flow of a non-Newtonian fluid in the presence of thermal radiation: An analytic solution, Zeitschrift für Naturforschung A. A J. Phys. Sci. 67 (3–4), 147–152.

Khan, U., Ahmed, N., Khan, S.I.U., Bano, Saima, Mohyud-din, S.T., 2014. Unsteady Squeezing flow of Casson fluid between parallel plates. World J. Model. Simul. 10 (4), 308–319.

Khan, U., Ahmed, N., Khan, S.I.U., Zaidi, Z.A., Yang, X.J., Mohyud-Din, S.T., 2014a. On unsteady two-dimensional and axisymmetric squeezing flow between parallel plates. Alexandria Eng. J. 53, 463–468.

Khan, U., Ahmed, N., Zaidi, Z.A., Asadullah, M., Mohyud-Din, S.T., 2014b. MHD squeezing flow between two infinite plates. Ain Shams Eng. J. 5, 187–192.

Khan, U., Ahmed, N., Khan, S.I.U., Mohyud-din, S.T., 2014c. Thermo-diffusion effects on MHD stagnation point flow towards a stretching sheet in a nanofluid. Propul. Power Res. 3 (3), 151–158

Kuzma, D.C., 1968. Fluid inertia effects in squeeze films. Appl. Sci. Res. 18, 15–20.

Laun, H.M., Rady, M., Hassager, O., 1999. Analytical solutions for squeeze flow with partial wall slip. J. Non-Newtonian Fluid Mech. 81, 1–15.

McDonald, D.A., 1974. Blood Flows in Arteries, second ed. Arnold, London.

Mohyud-Din, S.T., Noor, M.A., Waheed, A., 2009. Variation of parameter method for solving sixth-order boundary value problems. Commun. Korean Math. Soc. 24, 605–615.

Mrill, E.W., Benis, A.M., Gilliland, E.R., Sherwood, T.K., Salzman, E.W., 1965. Pressure flow relations of human blood hollow fibers at low flow rates. J. Appl. Physiol. 20, 954–967.

Mustafa, M., Hayat, T., Obaidat, S., 2012. On heat and mass transfer in the unsteady squeezing flow between parallel plates. Meccanica 47, 1581–1589.

Nadeem, S., Haq, R.U., Lee, C., 2012. MHD flow of a Casson fluid over an exponentially shrinking sheet. Scientia Iranica 19, 1150–1553.

Nadeem, S., Haq, R.U., Akbar, N.S., Khan, Z.H., 2013. MHD three dimensional flow of Casson fluid past a porous linearly Stretching sheet. Alexandria Eng. J. 52, 577–582.

Nadeem, S., Mehmood, R., Akbar, N.S., 2014a. Optimized analytical solution for oblique flow of a Casson-nano fluid with convective boundary conditions. Int. J. Thermal Sci. 78, 90–100.

Nadeem, S., Haq, R.U., Akbar, N.S., 2014b. MHD three dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with convective boundary condition. IEEE Trans. Nanotechnol. 13, 109–115.

Nadeem, S., Mehmood, R., Akbar, N.S., 2014c. Oblique stagnation point flow of a Casson-Nano fluid towards a stretching surface with heat transfer. J. Comput. Theor. Nanosci. 11, 1422–1432.

Nakamura, M., Sawada, T., 1987. Numerical study on the laminar pulsatile flow of slurries. J. Non-Newtonian Fluid Mech. 22, 191–206.

Nakamura, M., Sawada, T., 1988. Numerical study on the flow of a Non-Newtonian fluid through an axisymmetric stenosis. J. Biomech. Eng. 110, 137–143.

Nhan, P.T., 2000. Squeeze flow of a viscoelastic solid. J. Non-Newtonian Fluid Mech. 95, 343–362.

Noor, M.A., Mohyud-Din, S.T., 2007. Variational iteration technique for solving higher order boundary value problems. Appl. Math. Comp. 189, 1929–1942.

Noor, M.A., Mohyud-Din, S.T., Waheed, A., 2008. Variation of parameters method for solving fifth-order boundary value problems. Appl. Math. Inf. Sci 2, 135–141.

Rashidi, M.M., Shahmohamadi, H., Dinarvand, S., 2008. Analytic approximate solutions for unsteady two-dimensional and axisymmetric squeezing flows between parallel plates. Math. Prob. Eng., 1–13.

Reynolds, O., 1886. On the theory of lubrication and its application to Mr Beauchamp Tower's experiments, including an experimental determination of the viscosity of olive oil. Philos. Trans. Royal Soc. London 177, 157–234.

Akber, N.S., Khan, Z.H., 2015. Heat transfer analysis of bi-viscous ciliary motion fluid. Int. J. Biomath. 8 (2), 13. http://dx.doi.org/10.1142/S1793524515500266.

Siddiqui, A.M., Irum, S., Ansari, A.R., 2008. Unsteady squeezing flow of a viscous MHD fluid between parallel plates, a solution using the homotopy perturbation method. Math. Model. Anal. 13, 565–576.

Stefan, M.J., 1874. Versuch Über die scheinbare adhesion'', Sitzungsberichte der Akademie der Wissenschaften in Wien. Mathematik-Naturwissen 69, 713–721.

Tichy, J.A., Winer, W.O., 1970. Inertial considerations in parallel circular squeeze film bearings. J. Lubr. Technol. 92, 588–592.

Usha, R., Sridharan, R., 1996. Arbitrary squeezing of a viscous fluid between elliptic plates. Fluid Dyn. Res. 18, 35–51.

Wang, C.Y., 1976. The squeezing of fluid between two plates. J. Appl. Mech. 43 (4), 579–583.

Wang, C.Y., Watson, L.T., 1979. Squeezing of a viscous fluid between elliptic plates. Appl. Sci. Res. 35, 195–207.

Wolfe, W.A., 1965. Squeeze film pressures. Appl. Sci. Res. 14, 77–90.

Yang, K.T., 1958. Unsteady laminar boundary layers in an incompressible stagnation flow. J. Appl. Math. Trans. ASME 80, 421–427.

Zaidi, Z.A., Jan, S.U., Ahmed, N., Khan, U., Mohyud-Din, S.T., 2013. Variation of parameters method for thin film flow of a third grade fluid down an inclined plane. Italian J. Pure Appl. Math. 31, 161–168.